

Closing today: 3.10

Closing Mon: 4.1(1) and 4.1(2)

Closing Wed: 4.3

Entry Task: (directly from homework)

Find the critical numbers for

$$y = x^3 + 3x^2 - 72x$$

4.1: Max/Min

Let $y = f(x)$.

When encountering a function
always ask:

1. What is the domain?
2. What are the “critical numbers”?

A **critical number** is a number $x = a$
that is in the domain and either

- (a) $f'(a) = 0$, or
- (b) $f'(a)$ does not exist.

Example:

1. $f(x) = 4x + \frac{1}{x}$

- a) What is the domain?
- b) What are the critical numbers?

2. $g(x) = 3x - x^{1/3}$

- a) What is the domain?
- b) What are the critical numbers?

Peaks and valleys?

The **absolute max** (or global max) is the highest overall point.

The **absolute min** (or global min) is the lowest overall point.

Big huge key awesome observation:

(Extreme Value Theorem)

Absolute max/min always occur at critical numbers or endpoints!

Easy procedure to answer ALL absolute max/min questions:

1. Find critical numbers.
2. Plug endpoints and critical numbers into the function.

Example: (like homework)

1. Find the absolute max and min of $f(x) = x^3 + 3x^2$ on the interval $[-1,2]$.

Small Note: The **value** of a function, $y = f(x)$, is the output y-value. A question asking for the absolute max of a function is asking for the **y-value**.

2. Find the abs. max and min of $f(x) = x \ln(x)$ on the interval $[1, e]$.

3. Find the absolute max and min of $f(x) = x\sqrt{1-x}$ on the interval $[-1, 1]$.

4.3 Classifying Critical Points

Recall:

$y = f(x)$	$y' = f'(x)$
horiz. tangent	zero
increasing	positive
decreasing	negative
vertical tangent, sharp corner, or not continuous	does not exist

Key, big, essential observation

Let $y = f(x)$ have a critical number at $x = a$; if $f'(x)$ changes from...

1. ...positive to negative, then a **local maximum** occurs at $x = a$.
2. ...negative to positive, then a **local minimum** occurs at $x = a$.

This is called the first derivative test.

Example:

1. Find and classify the critical numbers for

$$y = x^3 + 3x^2 - 72x$$

2. Find and classify the critical numbers of

$$y = x^4 - 2x^3$$

3. Find and classify the critical numbers of

$$y = x^{2/3}$$

4. Find and classify the critical numbers of

$$y = \frac{x^3}{x^2 - 1}$$